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Department of Electrical Engineering and Computer Science

6.685 電機機械 Electric Machines

課堂講義 4：基本同步機模型 2005 年 9 月 5 日
Class Notes 4: Elementary Synchronous Machine Models September 14, 2005

1. 導論

同步機是一種主要的電機機械，本章目標是導出一個簡單而有實際意義的同步機模型。我們可以從幾個不同方向來探討這個模型，這有助於對機器分析的了解，特別是當某一解析方法比其他方法來得適當時。我們將探討操作原理和容量計算。

對於機器的繞組，我們將從兩個觀點來推演，其一我們考慮繞組有近似於正弦分布的電流和磁交鏈，其二是集中繞組的觀點，並將其一般化為比較實際而且有用的繞組模型。

1 Introduction

The objective here is to develop a simple but physically meaningful model of the synchronous machine, one of the major classes of electric machine. We can look at this model from several different directions. This will help develop an understanding of analysis of machines, particularly in cases where one or another analytical picture is more appropriate than others. Both operation and sizing will be of interest here.

Along the way we will approach machine windings from two points of view. On the one hand, we will approximate windings as sinusoidal distributions of current and flux linkage. Then we will take a concentrated coil point of view and generalize that into a more realistic and useful winding model.

2. 示意圖：電流薄層

如以下簡單圖示，「機器」包含一個圓柱型轉子和一個同軸的圓筒型定子，兩者表面（轉子的外表面和定子的內表面）有正弦分佈的電流。

2 Physical Picture: Current Sheet Description

Consider this simple picture. The ‘machine’ consists of a cylindrical rotor and a cylindrical stator which are coaxial and which have sinusoidal current distributions on their surfaces: the outer surface of the rotor and the inner surface of the stator.

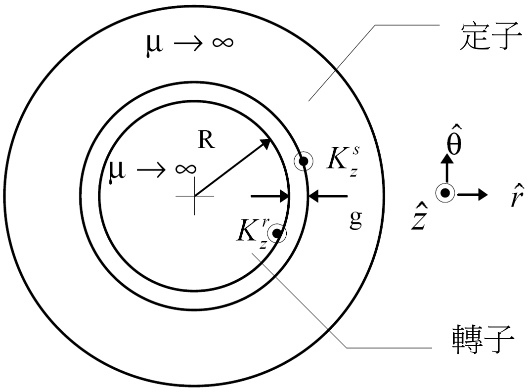

圖 1：基本機器模型：軸向圖

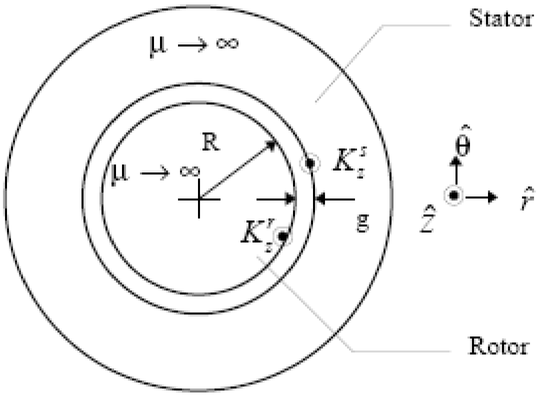

Figure 1: Elementary Machine Model: Axial View

轉子和定子都使用高導磁性材料做成（此處先假設導磁係數為無限大，稍後再詳細研究此問題），同時假設轉子和定子面向 z 軸方向的電流分布為正弦形狀：

The ‘rotor’ and ‘stator’ bodies are made of highly permeable material (we approximate this as being infinite for the time being, but this is something that needs to be looked at carefully later). We also assume that the rotor and stator have current distributions that are axially (z) directed and sinusoidal:

$$\begin{aligned} K_z^S &= K_S \cos p\theta \\ K_z^R &= K_R \cos p\left(\theta - \phi\right) \end{aligned}$$

式中ϕ是轉子位置的機械角度。轉子的電流分布對應於轉子是固定的。假設氣隙的大小 g 比半徑小很多：$g<<R$，則徑向磁通密度 B_r 在氣隙內可視為近乎均勻(亦即它不是半徑的函數)，並且符合：

Here, the angle φ is the physical angle of the rotor. The current distribution on the rotor is fixed with respect to the rotor. Now: assume that the air-gap dimension g is much less than the radius: g << R. It is not difficult to show that with this assumption the radial flux density Br is nearly uniform across the gap (i.e. not a function of radius) and obeys:

$$\frac{\partial B_r}{\partial R\theta} = -\mu_0 \frac{K_z^S + K_z^R}{g}$$

徑向磁通密度即為：

Then the radial magnetic flux density for this case is simply:

$$B_r = -\frac{\mu_0 R}{pg}\left(K_S \sin p\theta + K_R \sin p\left(\theta - \phi\right)\right)$$

接著可以計算轉子和定子表面的牽引力，因為表面電流分佈等於角向磁場：在定子表面為 $H_\theta = -K_z^S$ ；在轉子表面為 $H_\theta = K_z^R$ ，所以在轉子表面的牽引力為：

Now it is possible to compute the traction on rotor and stator surfaces by recognizing that the surface current distributions are the azimuthal magnetic fields: at the surface of the stator, $H_\theta = -K_z^S$, and at the surface of the rotor, $H_\theta = K_z^R$. So at the surface of the rotor, traction is:

$$\tau_\theta = T_{r\theta} = -\frac{\mu_0 R}{pg}\left(K_S \sin p\theta + K_R \sin p\left(\theta - \phi\right)\right) K_R \cos p\left(\theta - \phi\right)$$

平均值為：

The average of that is simply:

$$< \tau_\theta >= -\frac{\mu_0 R}{2pg} K_S K_R \sin p\phi$$

利用相同方法可以計算定子表面牽引力，結果與上式相同，但是符號相反。然後可以算得轉矩為：

The same exercise done at the surface of the stator yields the same results (with opposite sign). To find torque, use:

$$T = 2\pi R^2 \ell < \tau_\theta >= \frac{\mu_0 \pi R^3 \ell}{pg} K_S K_R \sin p\phi$$

整理上列討論，得：

We pause here to make a few observations:

1. 若知道表面電流 K_S 和 K_R，則轉矩是機器尺寸的三次方。表示可得到的剪力強度對機器尺寸之比為固定值，機器轉矩密度對於機器體積之比值也是固定值。

For a given value of surface currents Ks and Kr, torque goes as the third power of linear dimension. That implies that the achieved shear stress is constant with machine size. And the ratio of machine torque density to machine volume is constant.

2. 另一方面，如果氣隙為固定，轉矩是機器尺寸的 4 次方。因為機器體積是尺寸的 3 次方，這表示轉矩能力是機器體積的 4/3 次方。

If, on the other hand, gap is held constant, torque goes as the fourth power of machine volume. Since the volume of the machine goes as the third power, this implies that torque capability goes as the 4/3 power of machine volume.

3. 事實上，這並未充分說明全部。因假設表面電流密度是容積電流密度乘以繞組厚度，而繞組厚度可認為隨著機器尺寸而增加。當機器的半徑變大時，定子和轉子的表面電流密度也會增加，以致機器轉矩密度和功率密度趨向於增加得比機器體積要稍快一些。

Actually, this understates the situation since the assumed surface current densities are the products of volume current densities and winding depth, which one would expect to increase with machine size. As machine radius grows one would expect both stator and rotor surface current densities to grow. Thus machine torque (and power) densities tend to increase somewhat faster than linearly with machine volume.

4. 定子和轉子的電流分佈希望互相對齊。實際情況，定子的電流分佈並非靜止，而是依照下式在空間旋轉：

The current distributions want to align with each other. In actual practice what is done is to generate a stator current distribution which is not static as implied here but which rotates in space:

$$K_z^S = K_S \cos(p\theta - \omega t)$$

同時它拉著轉子一起旋轉。

and this pulls the rotor along.

5. 每一組定子和轉子電流分佈都能產生一個持續的最大轉矩，一旦加於轉子的轉矩小於該值，轉子即會自動調整到正確的角度。

For a given pair of current distributions there is a maximum torque that can be sustained, but as long as the torque that is applied to the rotor is less than that value the rotor will adjust to the correct angle.

3. 近似於連續的繞組形式

此時暫時還無法精確的依照實際繞組來計算表面電流，但是可以考慮繞組匝數的分佈近似於：

3 Continuous Approximation to Winding Patterns:

Now let’s try to produce those surface current distributions with physical windings. In fact we can’t do exactly that yet, but we can approximate a physical winding with a turns distribution that would look like:

$$\begin{aligned} n_S &= \frac{N_S}{2R}\cos p\theta \\ n_R &= \frac{N_R}{2R}\cos p(\theta - \phi) \end{aligned}$$

N_S 和 N_R 分別是定子和轉子繞組的總匝數，即：

Note that this implies that N_S and N_R are the total number of turns on the rotor and stator. i.e.:

$$p\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} n_S R d\theta = N_S$$

表面電流密度如前述的假設，其值：

Then the surface current densities are as we assumed above, with:

$$K_S = \frac{N_S I_S}{2R} \qquad K_R = \frac{N_R I_R}{2R}$$

到此處並沒有什麼特別不同之處。但是有了匝數，我們可以計算電感。切記這樣假設的繞組分佈意為：它們是定子和轉子表面的導線密度。正值表示導線在+z的方向；負值則表示導線在-z 的方向。亦即，若是繞組的端子電流為正，則 n

為正值時電流在+z 的方向，而 n 為負值時電流在-z 的方向。事實上，繞組是由許多基本線圈所組成，這些線圈的一半（走向為負的一半）與另一半（走向為正）相距*π/p* 的角度，與基本線圈交鏈的磁通乃為：

So far nothing is different, but with an assumed number of turns we can proceed to computing inductances. It is important to remember what these assumed winding distributions mean: they are the density of wires along the surface of the rotor and stator. A positive value implies a wire with sense in the +z direction, a negative value implies a wire with sense in the -z direction. That is, if terminal current for a winding is positive, current is in the +z direction if n is positive, in the -z direction if n is negative. In fact, such a winding would be made of elementary coils with one half (the negatively going half) separated from the other half (the positively going half) by a physical angle of *π/p*. So the flux linked by that elemental coil would be:

$$\Phi_i(\theta) = \int_{\theta-\pi/p}^{\theta} \mu_0 H_r(\theta') \ell R d\theta'$$

若只有定子的繞組受到激磁，徑向的磁場為：

So, if only the stator winding is excited, radial magnetic field is:

$$H_r = -\frac{N_S I_S}{2gp} \sin p\theta$$

所以基本的線圈磁通為

and thus the elementary coil flux is:

$$\Phi_i(\theta) = \frac{\mu_0 N_S I_S \ell R}{p^2 g} \cos p\theta$$

上式是和一個基本線圈交鏈的磁通。將所有基本線圈的磁交鏈「相加」，即得整個繞組的磁交鏈。在連續繞組的情況，此相當於對分佈的線圈做積分

Now, this is flux linked by an elementary coil. To get flux linked by a whole winding we must ‘add up’ the flux linkages of all of the elementary coils. In our continuous approximation to the real coil this is the same as integrating over the coil distribution:

$$\lambda_S = p \int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \Phi_i(\theta) n_S(\theta) R d\theta$$

此式等於

This evaluates fairly easily to:

$$\lambda_S = \mu_0 \frac{\pi}{4} \frac{\ell R N_S^2}{g p^2} I_s$$

上式表示定子繞組的自感為：

which implies a self-inductance for the stator winding of:

$$L_S = \mu_0 \frac{\pi}{4} \frac{\ell R N_S^2}{g p^2}$$

相同的方法可以導出轉子繞組的自感（需適當的修改空間變數），其方程式為：
The same process can be used to find self-inductance of the rotor winding (with appropriate changes of spatial variables), and the answer is:

$$L_R = \mu_0 \frac{\pi}{4} \frac{\ell R N_R^2}{g p^2}$$

計算兩繞組之間的互感，可以對其中一個繞組激磁，而算出和另一繞組交鏈的磁通。其方程式為：
To find the mutual inductance between the two windings, excite one and compute flux linked by the other. All of the expressions here can be used, and the answer is:

$$M(\phi) = \mu_0 \frac{\pi}{4} \frac{\ell R N_S N_R}{g p^2} \cos p\phi$$

接著，利用傳統的方法可以容易的算出轉矩。假設兩組繞組都激磁，磁輔能為：
Now it is fairly easy to compute torque using conventional methods. Assuming both windings are excited, magnetic coenergy is:

$$W'_m = \frac{1}{2} L_S I_S^2 + \frac{1}{2} L_R I_R^2 + M(\phi) I_S I_R$$

所以轉矩為：
and then torque is:

$$T = \frac{\partial W'_m}{\partial \phi} = -\mu_0 \frac{\pi}{4} \frac{\ell R N_S N_R}{g p} I_S I_R \sin p\phi$$

$N_S I_S$ 和 $N_R I_R$ 用下列值代入：

and then substituting for $N_S I_S$ and $N_R I_R$:

$$\begin{aligned} N_S I_S &= 2 R K_S \\ N_R I_R &= 2 R K_R \end{aligned}$$

則所求得的轉矩和磁場方法所求得的答案是一樣的：
we get the same answer for torque as with the field approach:

$$T = 2\pi R^2 \ell < \tau_\theta > = -\frac{\mu_0 \pi R^3 \ell}{p g} K_S K_R \sin p\phi$$

4. 傳統集總參數的同步機

此處我們將討論最簡單的多相同步機模型。這部同步機的轉子與前面所討論者相同，但是定子具有三組完全相同，而空間上以電機角 $120^o=2\pi/3$ 分離的繞組，這三組定子繞組具有相等的自感量(L_a)。

4 Classical, Lumped-Parameter Synchronous Machine:

Now we are in a position to examine the simplest model of a polyphase synchronous machine. Suppose we have a machine in which the rotor is the same as the one we were considering, but the stator has three separate windings, identical but with spatial orientation separated by an electrical angle of $120^\circ = 2\pi/3$. The three stator windings will have the same self-inductance (L_a).

三組定子繞組之間有互感，並具有 120^o 餘弦(cosine)的特徵。由於繞組之間的夾角都相同，

With a little bit of examination it can be seen that the three stator windings will have mutual inductance, and that inductance will be characterized by the cosine of 120°. Since the physical angle between any pair of stator windings is the same,

$$L_{ab} = L_{ac} = L_{bc} = -\frac{1}{2}L_a$$

轉子和各相定子繞組之間也有互感，以 M 代表其值：

There will also be a mutual inductance between the rotor and each phase of the stator. Using M to denote the magnitude of that inductance:

$$\begin{aligned}
M &= \mu_0 \frac{\pi}{4}\frac{\ell R N_a N_f}{g p^2} \\
M_{af} &= M \cos(p\phi) \\
M_{bf} &= M \cos\left(p\phi - \frac{2\pi}{3}\right) \\
M_{cf} &= M \cos\left(p\phi + \frac{2\pi}{3}\right)
\end{aligned}$$

根據講義第一章，此系統的轉矩為：

We show in Chapter 1 of these notes that torque for this system is:

$$T = -pMi_a i_f \sin(p\phi) - pMi_b i_f \sin\left(p\phi - \frac{2\pi}{3}\right) - pMi_c i_f \sin\left(p\phi + \frac{2\pi}{3}\right)$$

5. 平衡運轉

這部機器的運轉情況如下：轉子以固定速率旋轉，磁場電流保持固定，三相定子電流都是時間正弦波，波幅相等但相位相差 120 度，

5 Balanced Operation:

Now, suppose the machine is operated in this fashion: the rotor turns at a constant velocity, the field current is held constant, and the three stator currents are sinusoids in time, with the same amplitude and with phases that differ by 120 degrees.

$$
\begin{aligned}
p\phi &= \omega t + \delta_i \\
i_f &= I_f \\
i_a &= I\cos(\omega t) \\
i_b &= I\cos\left(\omega t - \frac{2\pi}{3}\right) \\
i_c &= I\cos\left(\omega t + \frac{2\pi}{3}\right)
\end{aligned}
$$

經過直接而冗長的計算，可以得到如下的轉矩：

Straightforward (but tedious) manipulation yields an expression for torque:

$$T = -\frac{3}{2}pMII_f\sin\delta_i$$

運轉在平衡電流、機械轉速與電源頻率($p\Omega=\omega$)相同的情況下，此機器的轉矩為固定值，相角δ_i稱為轉矩角，但是使用轉矩角時必須十分小心，因為有好幾個轉矩角。

Operated in this way, with balanced currents and with the mechanical speed consistent with the electrical frequency ($p\Omega=\omega$), the machine exhibits a constant torque. The phase angle δ_i is called the torque angle, but it is important to use some caution, as there is more than one torque angle.

從電源接線端子看機器內部，相 A 所交鏈的磁通為：

Now, look at the machine from the electrical terminals. Flux linked by Phase A will be:

$$\lambda_a = L_a i_a + L_{ab} i_b + L_{ac} i_c + MI_f\cos p\phi$$

注意，在平衡條件下，三相電流之和為零，並且相與相之間的互感相等，所以可以簡化為：

Noting that the sum of phase currents is, under balanced conditions, zero and that the mutual phase-phase inductances are equal, this simplifies to:

$$\lambda_a = (L_a - L_{ab})\, i_a + MI_f \cos p\phi = L_d i_a + MI_f \cos p\phi$$

式中使用 L_d 來代表同步電感。

如果此機器的旋轉速度和電源頻率一致，則稱之為同步運轉，可以使用複數來表示穩態的正弦函數：

where we use the notation L_d to denote synchronous inductance.

Now, if the machine is turning at a speed consistent with the electrical frequency we say it is operating synchronously, and it is possible to employ complex notation in the sinusoidal steady state. Then, note:

$$i_a = I\cos(\omega t + \theta_i) = \mathrm{Re}\left\{ I e^{j\omega t + \theta_i} \right\}$$

若磁通以複數表示：

If , we can write an expression for the complex amplitude of flux as:

$$\lambda_a = \mathrm{Re}\left\{ \underline{\Lambda}_a e^{j\omega t} \right\}$$

電流可以寫成：

where we have used this complex notation:

$$\begin{aligned} \underline{I} &= I e^{j\theta_i} \\ \underline{I}_f &= I_f e^{j\theta_m} \end{aligned}$$

此系統的端電壓為：

Now, if we look for terminal voltage of this system, it is:

$$v_a = \frac{d\lambda_a}{dt} = \mathrm{Re}\left\{ j\omega \underline{\Lambda}_a e^{j\omega t} \right\}$$

此系統可以描述如圖 2 之等效電路

This system is described by the equivalent circuit shown in Figure 2.

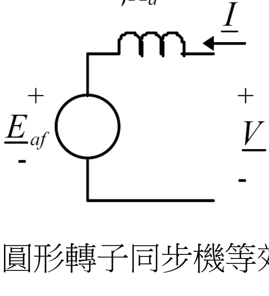

圖 2：圓形轉子同步機等效電路

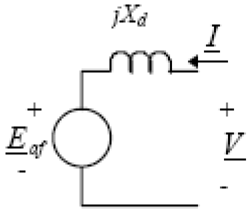

Figure 2: Round Rotor Synchronous Machine Equivalent Circuit

內部電壓為：

where the internal voltage is:

$$\underline{E}_{af} = j\omega M I_f e^{j\theta_m}$$

若此機器由電壓源供電(亦即電壓 V 固定)，端電流為：

Now, if that is connected to a voltage source (i.e. if V is fixed), terminal current is:

$$\underline{I} = \frac{\underline{V} - E_{af} e^{j\delta}}{jX_d}$$

式中 $X_d = \omega L_d$ 為同步電抗。

則 A 相的實功率和虛功率為：

where $X_d = \omega L_d$ is the synchronous reactance.

Then real and reactive power (in phase A) are:

$$\begin{aligned} P + jQ &= \frac{1}{2}\underline{V}\underline{I}^* \\ &= \frac{1}{2}\underline{V}\left(\frac{\underline{V} - E_{af}e^{j\delta}}{jX_d}\right)^* \\ &= \frac{1}{2}\frac{|\underline{V}|^2}{-jX_d} - \frac{1}{2}\frac{VE_{af}e^{j\delta}}{-jX_d} \end{aligned}$$

由此可得實功率和虛功率，分別為：

This makes real and reactive power:

$$\begin{aligned} P_a &= -\frac{1}{2}\frac{VE_{af}}{X_d}\sin\delta \\ Q_a &= \frac{1}{2}\frac{V^2}{X_d} - \frac{1}{2}\frac{VE_{af}}{X_d}\cos\delta \end{aligned}$$

三相總實功率乃為：

If we consider all three phases, real power is

$$P = -\frac{3}{2}\frac{VE_{af}}{X_d}\sin\delta$$

最後一點，這些機器可以當做電動機，也可以當做發電機運轉。

Now, at last we need to look at actual operation of these machines, which can serve either as motors or as generators.

當做電動機和發電機運轉的向量圖，分別示於圖 3 和圖 4。

Vector diagrams that describe operation as a motor and as a generator are shown in Figures 3 and 4, respectively.

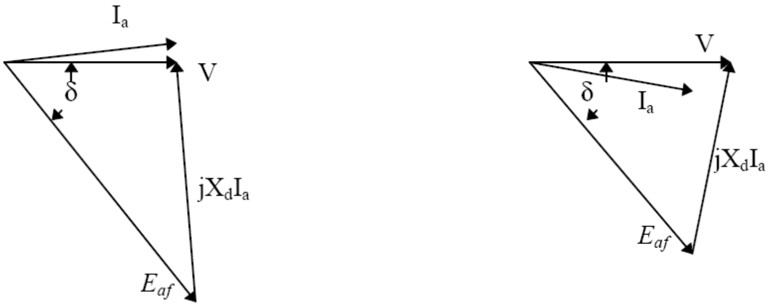

圖 3：電動機運轉，過激和欠激的情況

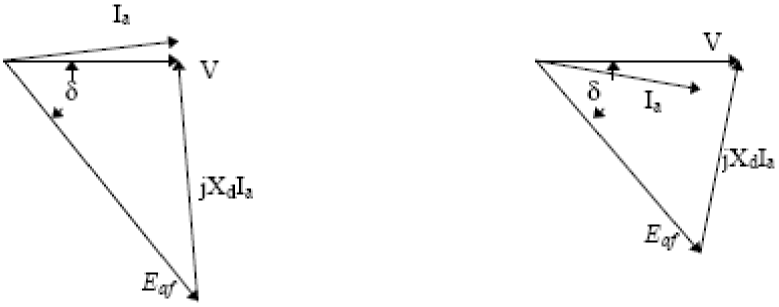

Figure 3: Motor Operation, Over- and Under- Excited

當做發電機運轉與當做電動機運轉並沒有多大差別，但是通常端電流的方向為反方向。

Operation as a generator is not much different from operation as a motor, but it is common to make notations with the terminal current given the opposite (“generator”) sign.

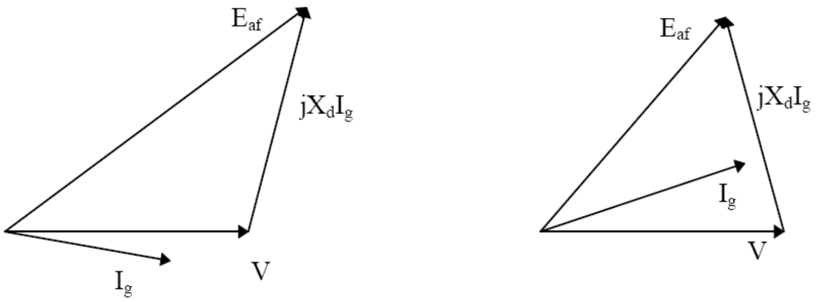

圖 4：發電機運轉，過激和欠激的情況

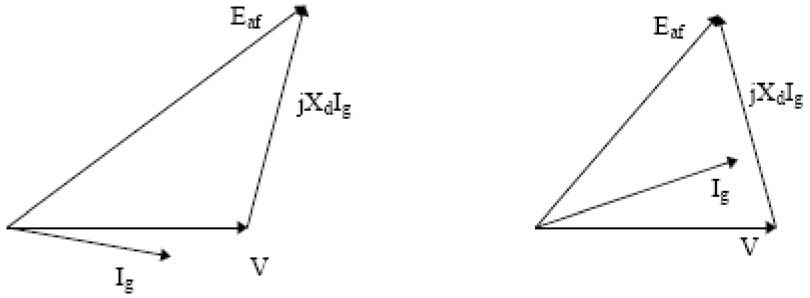

Figure 4: Generator Operation, Over- and Under- Excited

6. 模型的一致化

由前面的討論，知道預測同步機的功率及／或轉矩特性可由兩個方向進行，其一，由轉子和定子電流以及電力角來計算轉矩：

6 Reconciliation of Models

We have determined that we can predict its power and/or torque characteristics from two points of view : first, by knowing currents in the rotor and stator we could derive an expression for torque vs. a power angle:

$$T = -\frac{3}{2} p M I I_f \sin \delta_i$$

另外，從電路學的方向，可以導出功率的公式：

From a circuit point of view, it is possible to derive an expression for power:

$$P = -\frac{3}{2} \frac{V E_{af}}{X_d} \sin \delta$$

由於功率是轉矩乘以速率，這表示：

and of course since power is torque times speed, this implies that:

$$T = -\frac{3}{2}\frac{VE_{af}}{\Omega X_d}\sin\delta = -\frac{3}{2}\frac{pVE_{af}}{\omega X_d}\sin\delta$$

本節講義將整理這些公式，對其意義做進一步解釋，然後將這些簡化理論歸納為凸極機，以介紹電機機械的雙軸理論。

In this section of the notes we will, first of all, reconcile these notions, look a bit more at what they mean, and then generalize our simple theory to salient pole machines as an introduction to two-axis theory of electric machines.

6.1 轉矩角

圖 5 是同步電動機的運轉向量圖，它標示了正常運轉中定子和轉子的磁動勢和磁通在空間的位置。端子磁通定為「實數」，畫在水平位置。當作電動機運轉時，轉子落後δ角，所以轉子的磁通 MI_f 會落在圖上所顯示的位置。圖上也顯示了定子電流，以及定子電流和轉子之間的轉矩角δ_i。從定子磁通 L_dI 和端子磁通Λ_t之間畫一條連線，再畫一條虛線 OA 與其垂直，則 OA 的長度為：

6.1 Torque Angles:

Figure 5 shows a vector diagram that shows operation of a synchronous motor. It represents the MMF's and fluxes from the rotor and stator in their respective positions in space during normal operation. Terminal flux is chosen to be 'real', or occupy the horizontal position. In motor operation the rotor lags by angle δ, so the rotor flux MI_f is shown in that position. Stator current is also shown, and the torque angle between it and the rotor, δ_i is also shown. Now, note that the dotted line OA, drawn perpendicular to a line drawn between the stator flux L_dI and terminal flux Λ_t, has length:

$$|OA| = L_dI\sin\delta_i = \Lambda_t\sin\delta$$

端子電壓 $V = \omega\Lambda_t$，$Ea = \omega MI_f$ 以及 $X_d = \omega L_d$，直接代入得：

Then, noting that terminal voltage $V = \omega\Lambda_t$, $E_a = \omega MI_f$ and $X_d = \omega L_d$, straightforward substitution yields:

$$\frac{3}{2}\frac{pVE_{af}}{\omega X_d}\sin\delta = \frac{3}{2}pMII_f\sin\delta_i$$

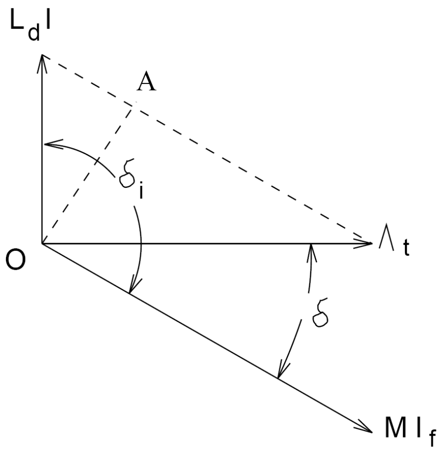

圖 5：同步機向量圖

$$\frac{3}{2}\frac{pVE_{af}}{\omega X_d}\sin\delta = \frac{3}{2}pMII_f\sin\delta_i$$

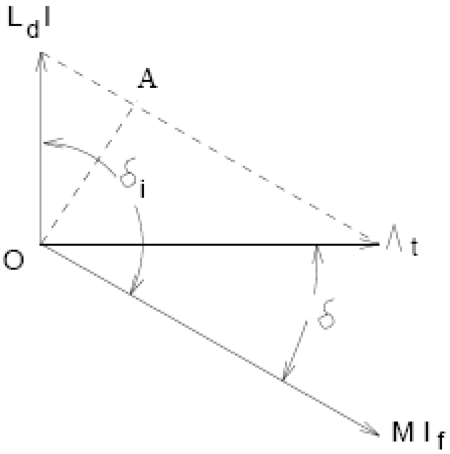

Figure 5: Synchronous Machine Phasor Addition

可知不論以電壓或以電流為基準，所繪的轉矩都會得到相同結果
So the current-and voltage-based pictures do give the same result for torque.

7. 標么系統

在下一節之前，讓我們先了解一下標么系統，這是一個很方便並且觀念上很有幫助的工具。基本概念很簡單：對於大部分的變數，我們可以指定一個基值，然後用變數的值除以基值，就得到該變數的標么值。一般而言，我們會將基值和實際運轉的量做一些關聯，例如，以機器的額定值作為電壓和電流的基值。若是這麼定義，則功率的基值就是：

7 Per-Unit Systems:

Before going on, we should take a short detour to look into per-unit systems, a notational device that, in addition to being convenient, will sometimes be conceptually helpful. The basic notion is quite simple: for most variables we will note a base quantity and then, by dividing the variable by the base we have a per-unit version of that variable. Generally we will want to tie the base quantity to some aspect of normal operation. So, for example, we might make the base voltage and current correspond with machine rating. If that is the case, then power base becomes:

$$P_B = 3V_B I_B$$

相同的，阻抗的基值就是：

and we can define, in similar fashion, an impedance base:

$$Z_B = \frac{V_B}{I_B}$$

此處請稍注意，上面是以線對中性點的電壓為電壓基值，以線電流為電流基值(兩者都是均方根值)。但並非一定要這麼定義。在三相系統中，我們也可以用線-對-線電壓作為基值，用三角接內的相電流作為電流基值：

Now, a little caution is required here. We have defined voltage base as line-neutral and current base as line current (both RMS). That is not necessary. In a three phase system we could very well have defined base voltage to have been line-line and base current to be current in a delta connected element:

$$V_{B\Delta} = \sqrt{3}V_B \qquad I_{B\Delta} = \frac{I_B}{\sqrt{3}}$$

在這種情況下，功率基值維持不變，但是阻抗基值差了 3 倍：

In that case the base power would be unchanged but base impedance would differ by a factor of three:

$$P_B = V_{B\Delta} I_{B\Delta} \qquad Z_{B\Delta} = 3Z_B$$

若將基值阻抗定義成與實際阻抗一致(請注意，三角接的阻抗 $3Z$ 等於 Y 接的 Z)，則一系統的標么阻抗與接線方式無關。事實上，這是標么系統的主要優點之一，相對於一般實際值有線-對-線、線-對-中性、均方根值、峰值…等等的變化，標么系統的每一個變數只有一個值。

However, if we were consistent with actual impedances (note that a delta connection of elements of impedance $3Z$ is equivalent to a wye connection of Z), the per-unit impedances of a given system are not dependent on the particular connection. In fact one of the major advantages of using a per-unit system is that per-unit values are uniquely determined, while ordinary variables can be line-line, line-neutral, RMS, peak, etc., for a large number of variations.

一般選擇線-對-線的值當做電壓基值和功率基值，則：
Perhaps unfortunate is the fact that base quantities are usually given as line-line voltage and base power. So that:

$$I_B = \frac{P_B}{\sqrt{3}V_{B\Delta}} \qquad Z_B = \frac{V_B}{I_B} = \frac{1}{3}\frac{V_{B\Delta}}{I_{B\Delta}} = \frac{V_{B\Delta}^2}{P_B}$$

通常我們用小寫字母作為標么變數：

Now, we will usually write per-unit variables as lower-case versions of the ordinary variables:

$$v = \frac{V}{V_B} \qquad p = \frac{P}{P_B} \quad \text{、、、 等等}$$

因此，使用標么系統，穩態運轉下的同步機實功率和虛功率可寫成：
Thus, written in per-unit notation, real and reactive power for a synchronous machine operating in steady state are:

$$p = -\frac{ve_{af}}{x_d}\sin\delta \qquad q = \frac{v^2}{x_d} - \frac{ve_{af}}{x_d}\cos\delta$$

當然，這是電動機的情況，而且是流進機器端子的實功率和虛功率。
These are, of course, in motor reference coordinates, and represent real and reactive power into the terminals of the machine.

8. 正常運轉

同步機可以隨時改變為電動機或者發電機運轉。當做電動機運轉時，它只在同步轉速時產生轉矩。渦輪發電機因為由原動機（例如蒸氣渦輪機）啟動，所以啟動方面沒有問題。一般的同步電動機就需要利用阻尼線圈（有時稱為啟動鼠籠）而以感應機模式來啟動。有了電子驅動裝置，此種機器即使在零轉速也可以視為同步。

8 Normal Operation:

The synchronous machine is used, essentially interchangeably, as a motor and as a generator. Note that, as a motor, this type of machine produces torque only when it is running at synchronous speed. This is not, of course, a problem for a turbo generator which is started by its prime mover (e.g. a steam turbine). Many synchronous motors are started as induction machines on their damper cages (sometimes called starting cages). And of course with power electronic drives the machine can often be considered to be “in synchronism” even down to zero speed.

不論當做電動機或者發電機，同步機都可以產生或者消耗虛功率。正常運轉情況下，實功率是由負載（電動機時）或者原動機（發電機時）所決定，而虛功率則由實功率和激磁電流來決定。
As either a motor or as a generator, the synchronous machine can either produce or consume reactive power. In normal operation real power is dictated by the load (if a motor) or the prime mover (if a generator), and reactive power is determined by the real power and by field current.

圖 6 是一種表示同步機容量的方法，此圖為發電機的運轉模式，所以 p 和 q 的符號反過來，但其他部分就和我們所預期的情況相同。我們依照一般計算結果而畫出 p 和 q ，從 $q = -v^2/x_d$ 的位置開始（請記得一般情況下 $v = 1$ 標么），則 p 和 q 的軌跡相當於一個長度為 ve_{af} / x_d 的向量掃過δ角度。此圖稱為容量表，是因為它能夠很清楚的看出同步機（此例為發電機）能夠達到的條件。共有三個清楚的容量限制。上方的限制是一個圓弧（向量的軌跡），稱之為磁場容量；第二個限制是由 $|p+jq|$ 所構成的圓弧，這當然和電樞電流的大小有關，因此稱為電樞容量；最後一個限制是機器的穩定度，是由於轉矩角不能超過 90 度而構成。實際上還有其他的限制可以在這圖上表示。例如，大型同步發電機在非常欠激的情況下（很大的負 q 值）運轉，通常會有定子鐵芯過熱的問題，因此另一個限制就是要避免在穩定度極限附近運轉；有些非常大的機器會有多於一個的冷卻狀態（例如不同的冷卻氫氣壓力），則上述的部分或全部極限值各有多條曲線。
Figure 6 shows one way of representing the capability of a synchronous machine. This picture represents operation as a generator, so the signs of p and q are reversed, but all of the other elements of operation are as we ordinarily would expect. If we plot p and q (calculated in the normal way) against each other, we see the construction at the right. If we start at a location $q = -v^2/x_d$, (and remember that normally $v = 1$ per-unit) , then the locus of p and q is what would be obtained by swinging a vector of length ve_{af}/x_d over an angle δ. This is called a capability chart because it is an easy way of visualizing what the synchronous machine (in this case generator) can do. There are three easily noted limits to capability. The upper limit is a circle (the one traced out by that vector) which is referred to as field capability. The second limit is a circle that describes constant $|p + jq|$. This is, of course, related to the magnitude of armature current and so this limit is called *armature* capability. The final limit is related to machine stability, since the torque angle cannot go beyond 90 degrees. In actuality there are often other limits that can be represented on this type of a chart. For

example, large synchronous generators typically have a problem with heating of the stator iron when they attempt to operate in highly under excited conditions (q strongly negative), so that one will often see another limit that prevents the operation of the machine near its stability limit. In very large machines with more than one cooling state (e.g. different values of cooling hydrogen pressure) there may be multiple curves for some or all of the limits.

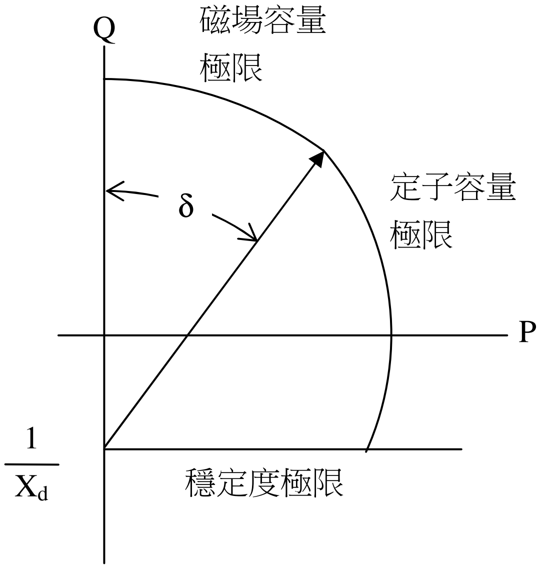

圖 6：同步發電機容量圖

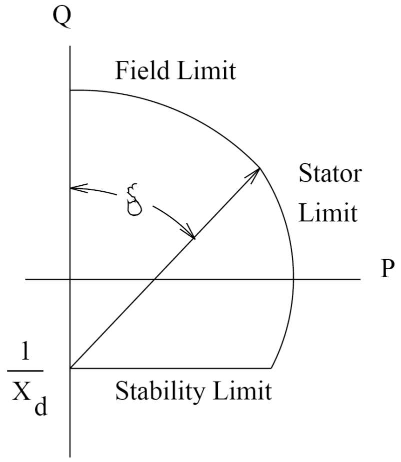

Figure 6: Synchronous Generator Capability Diagram

另一個描述同步機極限值的方法是使用 V 曲線，如圖 7 之例子。此種曲線圖標示電樞電流和磁場電流的關係。圖上直接可以看出磁場和電樞電流的極限值（就

是圖中右手邊和上面的邊界）。機器的穩定度極限值為每條曲線左上端終止點。值得注意的是，每條曲線的最小值是功率因數為 1 時。事實上，還有可能畫出另一種曲線圖，稱之為「複合曲線」，即針對某一固定功率因數繪出磁場電流對實功率的關係。

Another way of describing the limitations of a synchronous machine is embodied in the Vee Curve. An example is shown in Figure 7 . This is a cross-plot of magnitude of armature current with field current. Note that the field and armature current limits are straightforward (and are the right-hand and upper boundaries, respectively, of the chart). The machine stability limit is what terminates each of the curves at the upper left-hand edge. Note that each curve has a minimum at unity power factor. In fact, there is yet another cross-plot possible, called a compounding curve, in which field current is plotted against real power for fixed power factor.

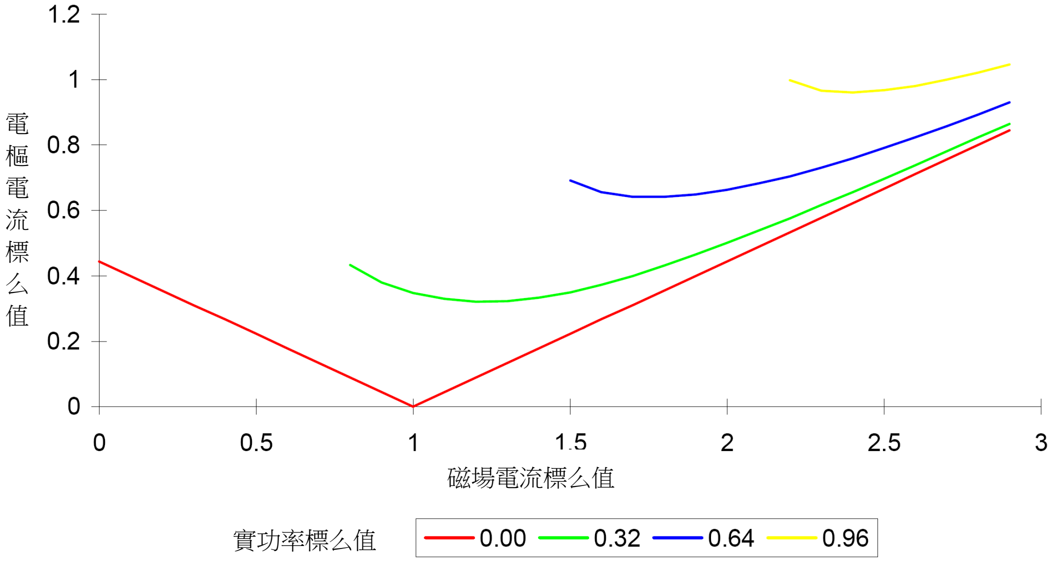

圖 7：同步機 V 曲線

9. 凸極機：雙反應理論

到目前我們已經介紹了所謂的「圓形轉子」同步機，圓形轉子同步機的定子感抗不受轉子位置的影響。在大型渦輪發電機和許多較小的二極同步機，這種理論相當合適，但是它卻不適用於許多同步電動機或者較低速的發電機，這些機器是將磁場線圈繞在鋼製的磁極上，然後利用螺栓或楔形榫頭將磁極固定到轉子上，以節省製造成本。但是這種設計所產生的磁場，各方向並不一致，影響了其運轉。所依循的原理是雙反應理論，並引申出旋轉磁場轉換而成為現代動態分析的基礎。

9 Salient Pole Machines: Two-Reaction Theory

So far, we have been describing what are referred to as “round rotor” machines, in which stator reactance is not dependent on rotor position. This is a pretty good approximation for large turbine generators and many smaller two-pole machines, but it is not a good approximation for many synchronous motors nor for slower speed generators. For many such applications it is more cost effective to wind the field conductors around steel bodies (called poles) which are then fastened onto the rotor body, with bolts or dovetail joints. These produce magnetic anisotropies into the machine which affect its operation. The theory which follows is an introduction to two-reaction theory and consequently for the rotating field transformations that form the basis for most modern dynamic analyses.

圖 8 是很簡單的凸極機示意圖，主要用來說明分析同步機的方法。與圓形轉子同步機一樣，定子繞組裝在由高導磁性材料做成的環形鐵芯表面線槽內，但磁場繞組則繞在鐵磁極外面。將定子電流薄層分成兩個成分：一個與磁場同方向；另一個和磁場成正交（90 度相位差）。請記住這兩個電流成分是由定子的相電流（線性）合成。而相電流能夠與 *d*-軸（直軸）和 *q*-軸（橫軸）電流直接轉換，本課程講義第 8 章將有說明。

Figure 8 shows a very schematic picture of the salient pole machine, intended primarily to show how to frame this analysis. As with the round rotor machine the stator winding is located in slots in the surface of a highly permeable stator core annulus. The field winding is wound around steel pole pieces. We separate the stator current sheet into two components: one aligned with and one in quadrature to the field. Remember that these two current components are themselves (linear) combinations of the stator phase currents. The transformation between phase currents and the d- and q-axis components is straightforward and will appear in Chapter 8 of these notes.

現在的重點是將磁動勢(MMF)和磁通分解成兩個互相垂直的成分，並假設每一個成分都成正弦波。這兩成分分別和機器的直軸與橫軸同方向。直軸與磁場繞組同方向，而橫軸則引前直軸 90 度。若 ϕ 代表直軸和 a 相軸線間之角度，則與 a 相交鏈的磁通可寫為：

The key here is to separate MMF and flux into two orthogonal components and to pretend that each can be treated as sinusoidal. The two components are aligned with the direct axis and with the quadrature axis of the machine. The direct axis is aligned with the field winding, while the quadrature axis leads the direct by90 degrees. Then, if φ is the angle between the direct axis and the axis of phase a, we can write for flux linking phase a:

$$\lambda_s = \lambda_d \cos\phi - \lambda_q \sin\phi$$

則，在穩態下運轉，若 $\mathrm{V}_a = \frac{d\lambda_a}{dt}$ ，並且 $\phi = \omega t + \delta$ ，

Then, in steady state operation, if $V_a = \frac{d\lambda_a}{dt}$ and $\phi = \omega t + \delta$,

$$V_a = -\omega\lambda_s \sin\phi - \omega\lambda_q \cos\phi$$

（譯註：式中等號右邊第一項似應為 $-\omega\lambda_d \sin\phi$ ）

由此可以定義：

which allows us to define:

$$\begin{aligned} V_d &= -\omega\lambda_q \\ V_q &= \omega\lambda_d \end{aligned}$$

可以想像為「電壓」向量引前「磁通（磁交鏈）」向量 90 度。

在線性區域，磁通（磁交鏈）為：

one might think of the ‘voltage’ vector as leading the ‘flux’ vector by 90 degrees. Now, if the machine is linear, those fluxes are given by:

$$\begin{aligned} \lambda_d &= L_d I_d + M I_f \\ \lambda_q &= L_q I_q \end{aligned}$$

一般而言，$L_d \neq L_q$。繞線式磁場同步機通常 $L_d > L_q$。大部分凸極永磁式(埋入式磁鐵)同步機則相反。

Note that, in general, $L_d \neq L_q$. In wound-field synchronous machines, usually $L_d > L_q$. The reverse is true for most salient (buried magnet) permanent magnet machines.

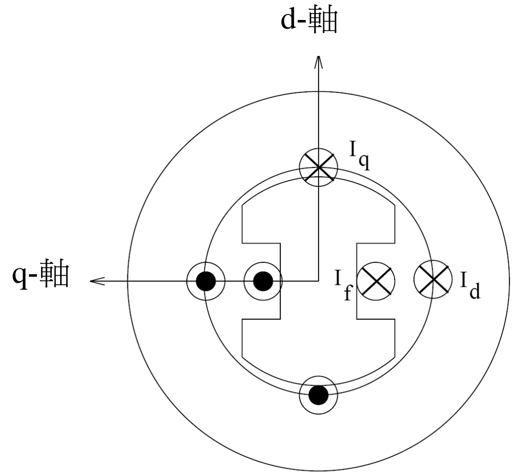

圖 8：凸極同步機示意圖

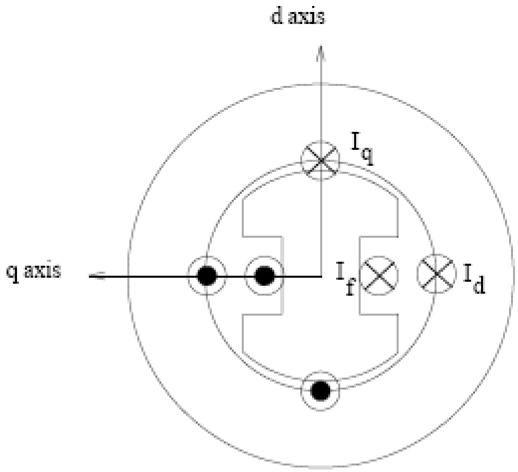

Figure 8: Cartoon of a Salient Pole Synchronous Machine

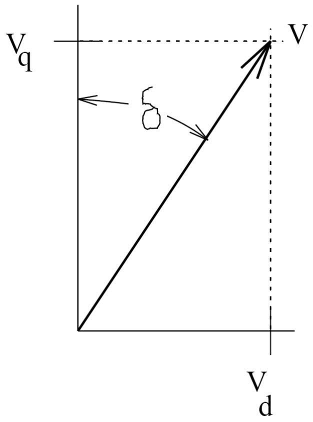

圖 9：端電壓的分解

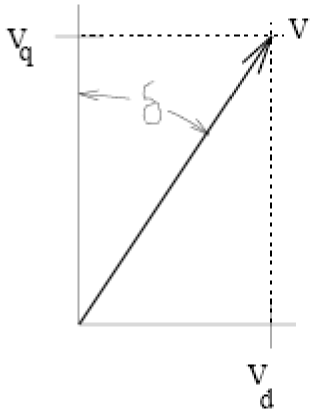

Figure 9: Resolution of Terminal Voltage

參考圖 9，端電壓可以分解為如下成分：

Referring to Figure 9, one can resolve terminal voltage into these components:

$$\begin{aligned} V_d &= V \sin \delta \\ V_q &= V \cos \delta \end{aligned}$$

或者寫成：

or

$$\begin{aligned} V_d &= -\omega \lambda_q = -\omega L_q I_q = V \sin \delta \\ V_q &= \omega \lambda_d = \omega L_d I_d + \omega M I_f = V \cos \delta \end{aligned}$$

上兩式可以容易導出：

which is easily inverted to produce:

$$\begin{aligned} I_d &= \frac{V \cos \delta - E_{af}}{X_d} \\ I_q &= -\frac{V \sin \delta}{X_q} \end{aligned}$$

式中

where

$$X_d = \omega L_d \qquad X_q = \omega L_q \qquad E_{af} = \omega M I_f$$

此處是以普通的變數來討論（本節說明可以顯現使用標么值的好處），每一個變數都代表其峰值。若是以複數考慮：

Now, we are working in ordinary variables (this discussion should help motivate the use of per-unit!), and each of these variables is peak amplitude. Then, if we take up a complex frame of reference:

$$\begin{aligned} \underline{V} &= V_d + j V_q \\ \underline{I} &= I_d + j I_q \end{aligned}$$

複數功率為：

complex power is:

$$P + jQ = \frac{3}{2} \underline{V I^*} = \frac{3}{2} \left\{ (V_d I_d + V_q I_q) + j \left(V_q I_d - V_d I_q \right) \right\}$$

或者：

or:

$$\begin{aligned} P &= -\frac{3}{2}\left(\frac{VE_{af}}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta\right) \\ Q &= \frac{3}{2}\left(\frac{V^2}{2}\left(\frac{1}{X_d}+\frac{1}{X_q}\right) - \frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\cos 2\delta - \frac{VE_{af}}{X_d}\cos\delta\right) \end{aligned}$$

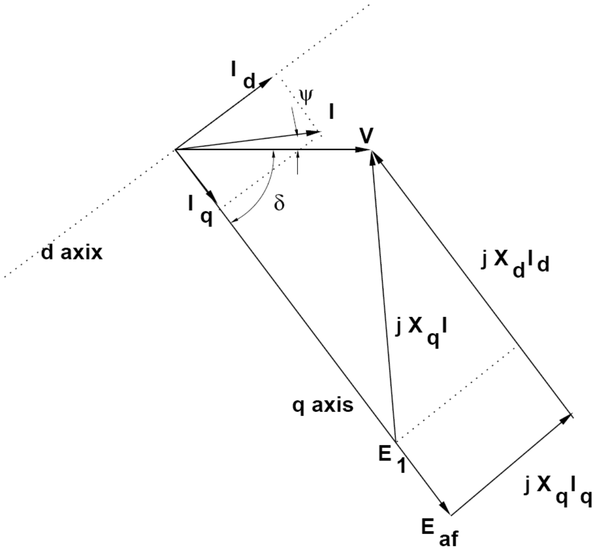

圖 10：向量圖：凸極機

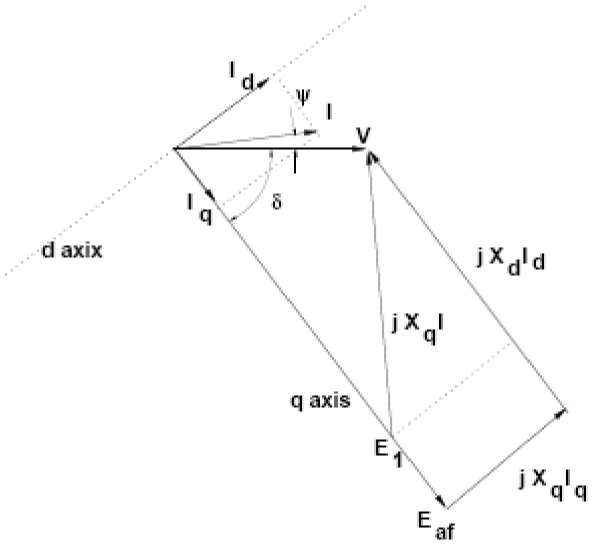

Figure 10: Phasor Diagram: Salient Pole Machine

圖 10 表示凸極機的向量圖，與圓形轉子同步機的向量圖稍有不同，在於定子電流被分解成 *d*-軸和 *q*-軸兩成份，同時這兩成份相關的電壓降也分別繪出。內部電壓 E_{af} 可以表示為：

A phasor diagram for a salient pole machine is shown in Figure 10. This is a little different from the equivalent picture for a round-rotor machine, in that stator current has been separated into its d-and q-axis components, and the voltage drops associated with those components have been drawn separately. It is interesting and helpful to recognize that the internal voltage E_{af} can be expressed as:

$$E_{af} = E_1 + (X_d - X_q)\, I_d$$

式中電壓 E_1 是在橫軸上。事實上，E_1 即為圓形轉子同步機的內電壓，其電抗為 X_q，並有相同定子電流和端電壓。操作點可以很容易的求得：

where the voltage E_1 is on the quadrature axis. In fact, E_1 would be the internal voltage of a round rotor machine with reactance X_q and the same stator current and terminal voltage. Then the operating point is found fairly easily:

$$\begin{aligned} \delta &= -\tan^{-1}\left(\frac{X_q I \cos\psi}{V + X_q I \sin\psi}\right) \\ E_1 &= \sqrt{(V + X_q I \sin\psi)^2 + (X_q I \cos\psi)^2} \end{aligned}$$

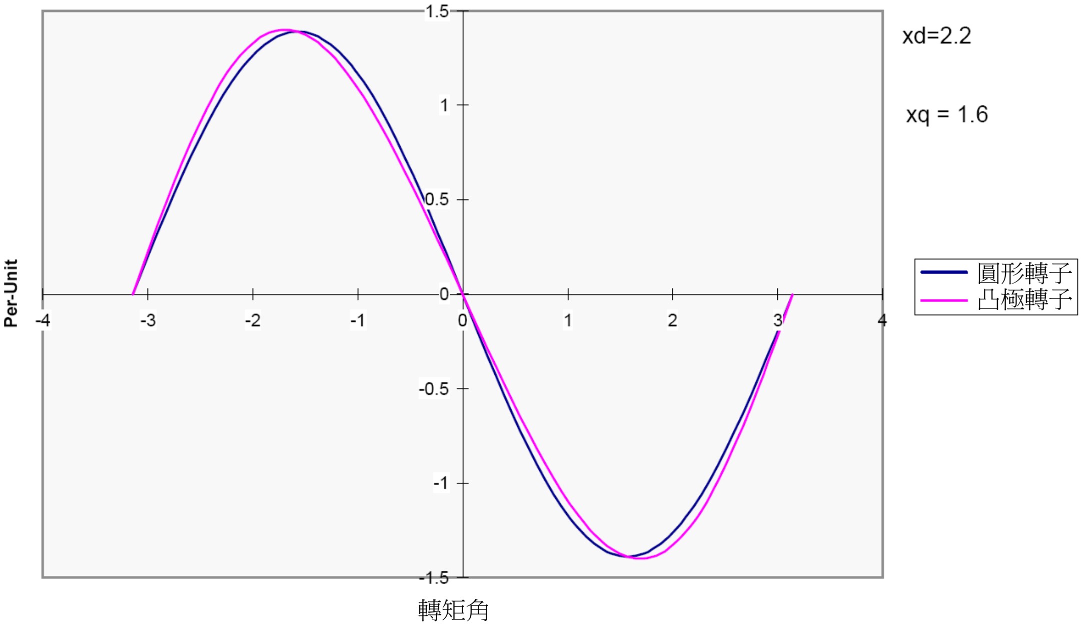

圖 11：轉矩角曲線：圓形轉子和凸極機

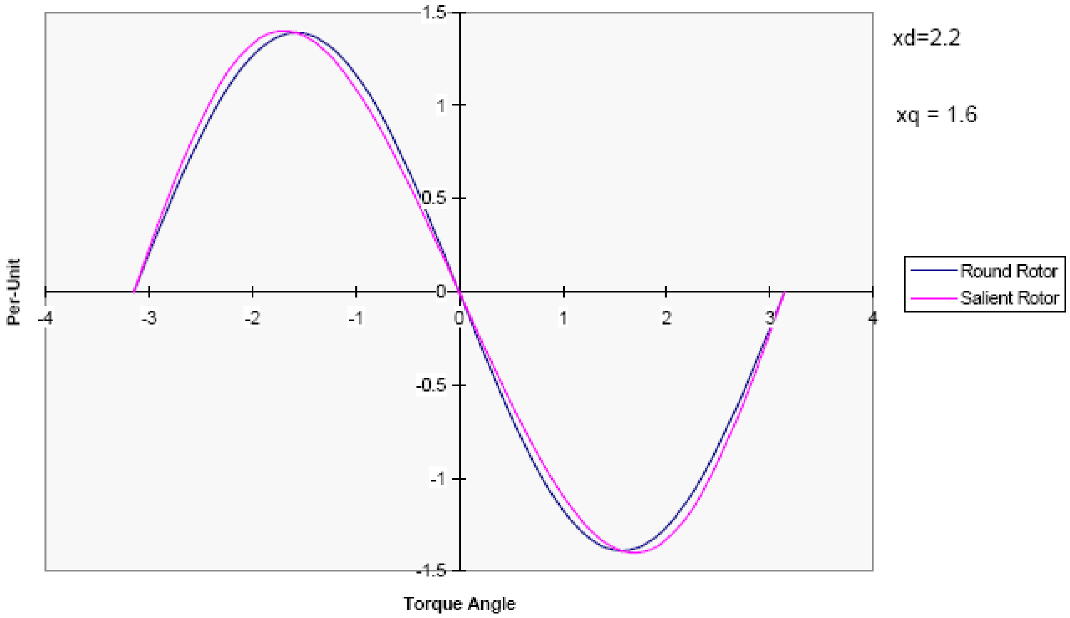

Figure 11: Torque-Angle Curves: Round Rotor and Salient Pole Machines

圖 11 顯示圓形轉子和凸極機轉矩角曲線的比較，由此圖可以了解為什麼人們在做諸如暫態穩定度這樣的電力系統分析時，常常忽略凸極的效應。

A comparison of torque-angle curves for a pair of machines, one with a round, one with a salient rotor is shown in Figure 11 . It is not too difficult to see why power systems analysts often neglect saliency in doing things like transient stability calculations.

10 額定值和尺寸的關係

利用在此之前所發展的簡單模型，已經能夠根據諸如磁通、表面電流密度等元素而建立機器尺寸和額定值之間的關係。首先，機器（電動機或發電機）的額定值是：

$$|P + jQ| = qVI$$

10 Relating Rating to Size

It is possible, even with the simple model we have developed so far, to establish a quantitative relationship between machine size and rating, depending (of course) on elements such as useful flux and surface current density. To start, note that the rating of a machine (motor or generator) is:

式中 q 代表相數；V 是每相電壓均方根值；I 為電流均方根值。欲建立機器額定值，得先建立電壓和電流，後面分別敘述之。

where *q* is the number of phases, *V* is the RMS voltage in each phase and *I* is the RMS current. To establish machine rating we must establish voltage and current, and we do these separately.

10.1 電壓

假設繞組的匝密度以正弦波形狀分佈：

10.1 Voltage

Assume that our sinusoidal approximation for turns density is valid:

$$n_a(\theta) = \frac{N_a}{2R}\cos p\theta$$

並假定工作磁通密度為：

And suppose that working flux density is:

$$B_r(\theta) = B_0 \sin p(\theta - \phi)$$

欲計算被繞組交鏈的磁通（以及後續計算電壓），先計算被線圈增量所交鏈的磁通：

Now, to compute flux linked by the winding (and consequently to compute voltage), we first compute flux linked by an incremental coil:

$$\lambda_i(\theta) = \int_{\theta}^{\theta+\frac{\pi}{p}} \ell B_r(\theta')Rd\theta'$$

所以，被整個線圈所交鏈的磁通為：

Then flux linked by the whole coil is:

$$\lambda_a = p\int_{-\frac{\pi}{2p}}^{\frac{\pi}{2p}} \lambda_i(\theta)n_a(\theta)Rd\theta = \frac{\pi}{4}\frac{2\ell RN_a}{p}B_0\cos p\phi$$

這是轉子在角 ϕ 時被交鏈的瞬時磁通。若機器以電機頻率 ω 運轉，並帶有某一相角，以致 $p\phi = \omega t + \delta$，端電壓的均方根值是：

This is instantaneous flux linked when the rotor is at angle φ. If the machine is operating at some electrical frequency ω with a phase angle so that pφ = ωt + δ, the RMS magnitude of terminal voltage is:

$$V_a = \frac{\omega}{p}\frac{\pi}{4}2\ell RN_a\frac{B_0}{\sqrt{2}}$$

最後一點請注意的是，有效的峰值電流密度受到機器線槽佔圓周的比例所限制：

Finally, note that the useful peak current density that can be used is limited by the fraction of machine periphery used for slots:

$$B_0 = B_s(1-\lambda_s)$$

式中 B_s 是機器線槽齒端的磁通密度，該密度受到磁性材料之飽和所限制。

where B_s is the flux density in the teeth, limited by saturation of the magnetic material.

10.2 電流

電流 I （均方根值）所產生的電流薄層（均方根值）為：

10.2 Current

The (RMS) magnitude of the current sheet produced by a current of (RMS) magnitude I is:

$$K_z = \frac{q}{2}\frac{N_a I}{2R}$$

反過來，電流值以電流薄層描述則為：

And then the current is, in terms of the current sheet magnitude:

$$I = 2RK_z\frac{2}{qN_a}$$

以面積電流密度 J_s、線槽空間因數 λ_s 以及線槽深度 h_s 表示，表面電流密度為：

Note that the surface current density is, in terms of area current density J_s, slot space factor λ_s and slot depth h_s:

$$K_z = \lambda_s J_s h_s$$

由此可得到以機器尺寸和有用的電流密度所描述的端電流：

This gives terminal current in terms of dimensions and useful current density:

$$I = \frac{4R}{qN_a}\lambda_s h_s J_s$$

10.3 額定值

將這些公式整理，機器額定值成為：

10.3 Rating

Assembling these expressions, machine rating becomes:

$$|P + jQ| = qVI = \frac{\omega}{p} 2\pi R^2 \ell \frac{B_s}{\sqrt{2}} \lambda_s (1 - \lambda_s) h_s J_s$$

此一公式很容易解釋。線槽因數乘以 1 減線槽因數之積很快就最佳化為 1/4(當λ_s = *1* 時) 。可解釋為

This expression is actually fairly easily interpreted. The product of slot factor times one minus slot factor optimizes rather quickly to 1/4 (when $\lambda_s = 1$). We could interpret this as:

$$|P = jQ| = A_s u_s \tau^*$$

式中互相作用之面積為：

where the interaction area is:

$$A_s = 2\pi R \ell$$

互相作用之表面速度為：

The surface velocity of interaction is:

$$u_s = \frac{\omega}{p} R = \Omega R$$

公式內「看起來像」是牽引力的片段為：

and the fragment of expression which “looks like” traction is:

$$\tau^* = h_s J_s \frac{B_s}{\sqrt{2}} \lambda_s (1 - \lambda_s)$$

請注意這並不全然是牽引力，因為電流和磁通不一定會很理想的同軸，這就是為什麼公式中會同時包含有效功和無效功。

Note that this is not quite traction since the current and magnetic flux may not be ideally aligned, and this is why the expression incorporates reactive as well as real power.

到此還並不完整。B_s 的極限值很容易了解是因磁性材料飽和所引起，其他剪力密度之重要成分，$h_s J_s$ 需要進一步解釋。

This is not quite yet the whole story. The limit on B_s is easily understood to be caused by saturation of magnetic material. The other important element on shear stress density, $h_s J_s$ is a little more involved.

10.4 電抗之角色

同步電抗標么值或者正常化值為：

10.4 Role of Reactance

The per-unit, or normalized synchronous reactance is:

$$x_d = X_d \frac{I}{V} = \frac{\mu_0 R}{pg} \frac{\lambda_s}{1-\lambda_s} \sqrt{2} \frac{h_s J_s}{B_s}$$

這本身看來有意思，但由此解答 $h_s J_a$ （譯註：似應為 $h_s J_s$）會變得有用：

While this may be somewhat interesting by itself, it becomes useful if we solve it for $h_s J_a$:

$$h_s J_a = x_d g \frac{p(1-\lambda_s) B_s}{\mu_0 R \lambda_s \sqrt{2}}$$

（譯註：等號左邊似應為 $h_s J_s$）

此即，若 x_d 固定，$h_s J_a$ （以及功率）直接和氣隙長度 g 有關。欲了解 g 的極限值，需先回答磁場繞組所產生的有效氣隙磁通能夠伸到多遠？要解此問題，需計算產生額定無載電壓所需的磁場電流，以及因為負載電流而需要增加的磁場電流。

That is, if x_d is fixed, $h_s J_a$ (and so power) are directly related to air-gap g. Now, to get a limit on g, we must answer the question of how far the field winding can “throw” effective air-gap flux? To understand this question, we must calculate the field current to produce rated voltage, no-load, and then the excess of field current required to accommodate load current.

在額定值下運轉，標么值磁場電壓為：

Under rated operation, per-unit field voltage is:

$$e_{af}^2 = v^2 + (x_d i)^2 + 2 x_d i \sin\psi$$

在額定值情況下，若 v 和 i 都是 1 pu，則

Or, if at rated conditions v and i are both unity (one per-unit), then

$$e_{af} = \sqrt{1 + x_d^2 + 2 x_d \sin\psi}$$

10.5 磁場繞組

若已知 x_d 和 ψ，以及標么值內部電壓 e_{af}，則計算需要的激磁電流可以先由估計「無載運轉」的磁場繞組電流著手：

10.5 Field Winding

Thus, given a value for x_d and ψ, per-unit internal voltage e_{af} is also fixed. Then field current required can be calculated by first estimating field winding current for "no-load operation".

$$B_r = \frac{\mu_0 N_f I_{fnl}}{2gp}$$

額定激磁電流為：

and rated field current is:

$$I_f = I_{fnl} e_{af}$$

或，需要的額定激磁電流為：

or, required rated field current is:

$$N_f I_f = \frac{2gp(1-\lambda_s)B_s}{\mu_0} e_{af}$$

其次，I_f 和磁場電流密度之關係可以表示為：

Next, I_f can be related to a field current density:

$$N_f I_f = \frac{N_{RS}}{2} A_{RS} J_f$$

式中 N_{RS} 是轉子的線槽數，而轉子線槽的面積 A_{RS} 為：

where N_{RS} is the number of rotor slots and the rotor slot area A_{RS} is

$$A_{RS} = w_R h_R$$

h_R 是轉子線槽高度，w_R 是轉子線槽寬度，

where h_R is rotor slot height and w_R is rotor slot width:

$$w_R = \frac{2\pi R}{N_{RS}} \lambda_R$$

則：

Then:

$$N_f I_f = \pi R \lambda_R h_R J_f$$

由此可求得氣隙寬度 g：

Now we have a value for air-gap g:

$$g = \frac{\pi}{2} \frac{\mu_0 R \lambda_R h_R J_f}{p(1-\lambda_s) B_s e_{af}}$$

進一步可以計算電樞表面電流密度：

This then gives us useful armature surface current density:

$$h_s J_s = \frac{\pi}{2\sqrt{2}} \frac{x_d}{e_{af}} \frac{\lambda_R}{\lambda_s} h_R J_f$$

至此已經介紹得差不多了。請注意 x_d/e_{af} 的比例可以相當小（如果電抗標么值小的話），在一般機器上從來就不會是很大的值，通常都小於 1。實際上，同步機的同步電抗很少大於 2 或 2.25 pu。這表示機器的轉子或者定子都可以成為剪應力密度（和額定值）的主要限制。最好的設計是使「兩者平衡」，也就是同時達到限制值。

We will not have a lot more to say about this. Note that the ratio of x_d/e_{af} can be quite small (if the per-unit reactance is small), will never be a very large number for any practical machine, and is generally less than one. As a practical matter it is unusual for the per-unit synchronous reatance of a machine to be larger than about 2 or 2.25 per-unit. What this tells us should be obvious: either the rotor or the stator of a machine can produce the dominant limitation on shear stress density (and so on rating). The best designs are “balanced”, with both limits being reached at the same time.